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OSCILLATIONS IN THE PARALLEL DISCHARGE OF TWO SUPERSONIC NONISOBARIC JETS

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Oscillating flows in systems of parallel supersonic nonisobaric jets discharged into a submerged space [1-12] are of considerable interest from the viewpoint of practical applications of this phenomenon and the development of models of transients in supersonic jets. Worth noting among the different studies in this area is [2], where for a system of parallel jets and jets parallel to a shield the author obtained the first data on the possible modes of vibration, their relative intensities, and the region of Mach numbers in which they exist. The authors of [4-12] presented the most complete empirical results on the modal composition of acoustic vibrations, their intensities, and the types of oscillations of the wave structure of the jets in relation to the discharge parameters and the interlaminar distance in two-jet systems. This information was compared with the analogous characteristics of single jets. However, no physical model has yet been presented to describe these characteristics in systems of jets.

In the present study, we obtain new empirical results for two parallel jets and propose models to describe the directionality of the acoustic radiation and the conditions for the excitation of oscillations.

1. Two parallel jets were created by means of supersonic conical nozzles with an outlet section having a diameter $d_a = 1.4 \cdot 10^{-2}$ m. We used nozzles with the Mach numbers M_a = 1, 1.5, 2, and 3.7 and the cone angle 9°. Compressed air from the prechamber was directed to smoothly change the distance between nozzles $\overline{S} = S/d_a$ from 1.8 to 7 and to displace the nozzles longitudinally relative to each other. We could remove one nozzle and in the middle of the inter-nozzle gap install a flat metal shield parallel to the jet axis. The shield measured 0.15 × 0.3 m.

The acoustic pressure pulsations were measured with two piezoelectric sensors. The diameter of their receiving part was $3 \cdot 10^{-3}$ m, while the limiting measurement frequency was 60 kHz. One sensor was placed on the line of centers of the nozzles midway between the latter. This sensor was positioned 3 diameters downflow of the nozzle edges. The constancy of the middle location of the sensor was assured by installing it at the center of a strip of rubber attached to the nozzles. To determine the directionality of the acoustic radiation in the plane perpendicular to the jets, the second sensor was positioned so that it could turn about its axis. This axis was parallel to the jets and passed through the center of the nozzle spacing at the level of the first sensor. The radius of rotation was 7 nozzle diameters.

Information on vibrations of the wave structure of the jets was obtained with IAB-451 shadowgraph equipped with an ISSh-15 stroboscopic lamp that was synchronized with the acoustic pressure pulsations. This made it possible to obtain images of the vibration phases with a high degree of averaging over random fluctuations of the flow field.

2. The first three schlieren photographs in Fig. 1 show the three main types of flows that were recorded experimentally. These flows exist during oscillations of two parallel jets in the system. The photographs correspond to the following discharge conditions: 1) $M_a = 1$, $n = 2.37$, $\overline{S} = 2.5$; 2) $M_a = 1$, $n = 1.83$, $\overline{S} = 5.15$; 3) $M_a = 1$, $n = 1.32$, $\overline{S} = 1.8$. It is evident from the first photograph that the wave structure of the jets undergoes flexural

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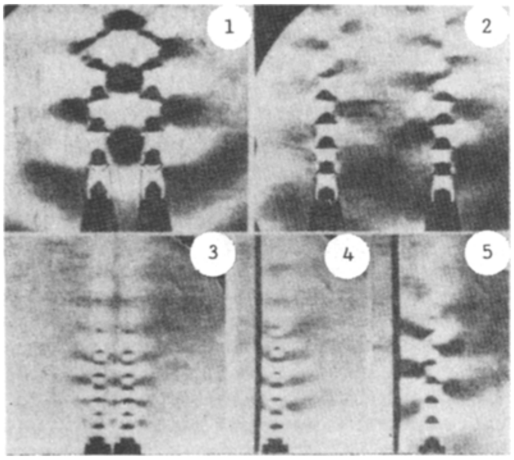

Fig. 1

oscillations which are symmetric relative to the plane dividing the nozzles. It was shown in [5] that the oscillations are polarized in a plane passing through the axis of the jet. The second photograph also shows flexural oscillations of the wave structure, but these oscillations are antisymmetric relative to the dividing plane. It is evident that the phase of the pressure pulsations in the external medium undergoes a sudden change of 180° in passing through the axis of the jet for both flows 1 and 2. The flow seen in the third photograph is characterized by an alternation of compression and tension of the cells of the wave structure and symmetry of the pressure waves in the external medium. It has been proposed [2, 5] that oscillations 1 and 2 are due to the development of a screw mode of instability of the mixing layer of the jets ($m = \pm 1$), while oscillations 3 are due to a symmetric mode of instability ($m = 0$). The presence of polarization in flows 1 and 2 is a manifestation of this instability in the two-jet system. An analysis of stroboscopic schlieren photographs of the flows with different M_a and $\bar{S}$ showed that vibrations 3 are realized for small Mach numbers of the theoretical jet ($M_t < 1.25$), while oscillations 1 and 2 are realized for $M_t >$ 1.25. This finding is consistent with the data in [2, 13] for single jets.

The spectra of the acoustic pressure pulsations accompanying the oscillatory process in the two-jet system contain two main modes. These were identified with screw modes C and B in [5, 7, 9, 13, 14]. Their frequency is determined by the parameters M_a and n, while the amplitude is also dependent on $\bar{S}$. Within the limits of accuracy of the measurements, the values of dimensionless frequency Sh coincide with the values presented for a single jet in [15] in the form of the generalized relations $Sh(M_t)$. Here, $Sh = f_d d_t/a$ (f_d is the frequency of the discrete tone, d_t is the diameter of the theoretical jet, and a is the speed of sound in the external medium). With a maximum error of 15%, the relations in [15] describe experimental measurements of the frequency of discrete tones of single jets in the range M_t = 1-3.5 and can be represented by the analytic expressions

$$Sh_C = 0{,}43\left(\sqrt{M_t^2-1}\right)^{-0{,}74},\ Sh_B = 0{,}375\left(\sqrt{M_t^2-1}\right)^{-0{,}62}. \quad (2.1)$$

Relations (2.1) give values of frequency close to those calculated with the approximate formula in [16] but also account for the presence of two frequency modes. It should be noted that the section of Eqs. (2.1) in the interval M_t = 1-1.25 probably corresponds to the group of frequency modes A_0, A_1, A_2 associated with the symmetric mode of instability [5, 7, 9, 13], since only oscillations of type 3 are seen in this range.

The discrete-tone amplitude characteristics recorded by the pressure-pulsation sensors are complex functions of the parameters M_a, n, and $\bar{S}$ and – for the moving sensor – the azimuthal angle θ. The situation is complicated by the fact that a change in the discharge parameters is accompanied by a change in the ratio of the amplitudes of the oscillation modes, with the energy of the oscillations changing from one mode to another (see [7, 9], for example). A similar effect is seen for single jets with the movement of a sound-reflecting disk along the nozzle and is connected with a change in the acoustic situation in the region of the jet [14, 17].

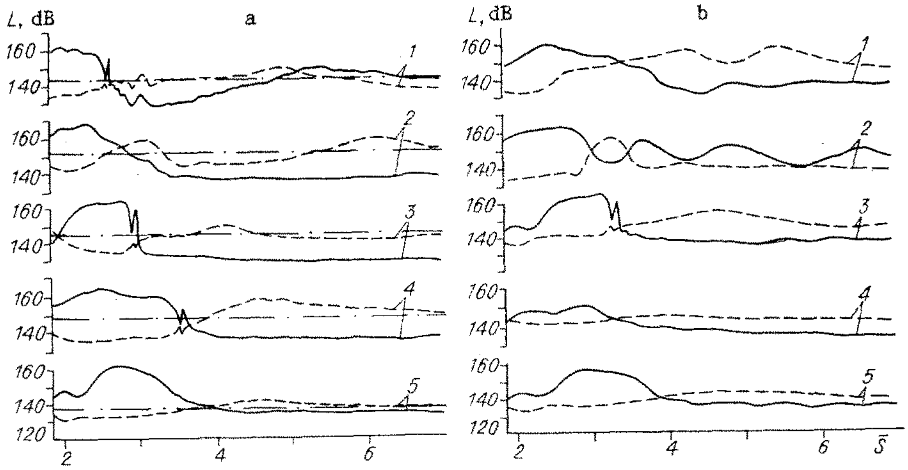

Fig. 2

Fig. 2a shows the dependence of the amplitude of the fundamental harmonic of the discrete tone for the stationary sensor on the nozzle spacing $\bar{S}$ (the solid lines correspond to mode B, the dashed lines to mode C) with the following discharge parameters: 1) $M_a = 1$, $n = 1.48$; 2) $M_a = 1$, $n = 1.83$; 3) $M_a = 1$, $n = 2.35$; 4) $M_a = 2$, $n = 0.57$; 5) $M_a = 2$, $n = 0.85$ (the dot-dash lines shows the level of pulsations recorded for one jet multiplied by a factor of $\sqrt{2}$). It is evident that for different M_a and n and small nozzle spacings, pulsation intensity in a system of parallel jets increases to significantly above the level created by two jets which are gasdynamically similar but acoustically independent. Along with the main intensity maximum on the graphs, there are less intensive increases in the level of the discrete tone. These increases lie within the region corresponding to large nozzle spacings.

Results on the maximum intensities of a discrete tone in a two-jet system when the parameters M_a, n, and $\bar{S}$ are varied were compared with data on the maximum intensities of the discrete tone of single jets in the presence of a sound-reflecting disk [15]. Using the coordinates M_a and n, Fig. 3 shows the region in which the amplitude of the discrete tone of a single jet (with mode discrimination) is at least 5 dB greater than the noise level in the same spectral range (the region was obtained by representing the graph in [15] in logarithmic coordinates). Intervals of degrees of underexpansion n are superimposed on the region. In these intervals, the level is more than 5 dB greater than in a system of two parallel jets (the dashed intervals show our data). It follows from the graph that the oscillation region in the two-jet system is no larger than the same region for a single jet. Comparison of the maximum ampltiudes of acoustic pulsation for one- and two-jet systems showed that the maximum level of pulsation in the two-jet system is no greater than the level created by a single jet with a disk for the same discharge parameters.

Figure 4 shows examples of measurements of the directionality of the radiation of the discrete tone in the plane of motion of the moving sensor. Curve 1 ($M_a = 1$, $n = 1.32$, $\bar{S} = 2.1$) corresponds to the flow pattern in photograph 3 in Fig 1, while curve 2 ($M_a = 1$, $n = 2.59$, $\bar{S} = 2.9$) corresponds to the pattern in photograph 1. It is evident that a multilobed direction diagram is obtained with the excitation of mode $m = 0$, while when $m = \pm 1$ we obtain a diagram with two lobes in the direction of the axis coincident with the center of the nozzles, i.e., in the direction of polarization of the oscillations of the wave structure of the jets. A similar pattern is obtained for the radiation of a dipole source and corresponds to axial symmetry of the perturbations seen in the photographs.

An acoustic field created by two jets with linear polarization of the wave-structure oscillations can be represented as a field of two interacting dipoles. Here and below, we will use the model representations in [16]. In accordance with the latter, the acoustic field of jets is formed by the radiation of dipole sources determined by the character of oscillation of the jets. Here, the direction diagram of the system of dipoles is described by the functions [18]

$$\cos\theta \cdot \sin\left(\frac{1}{2} kl \cos\theta\right) \qquad (2.2)$$

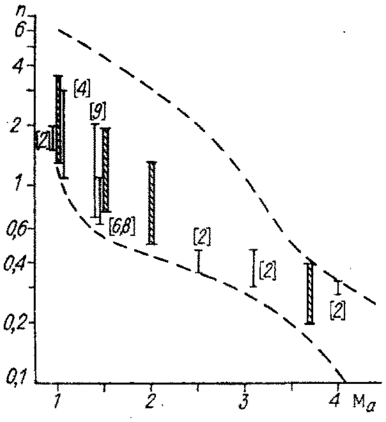

Fig. 3

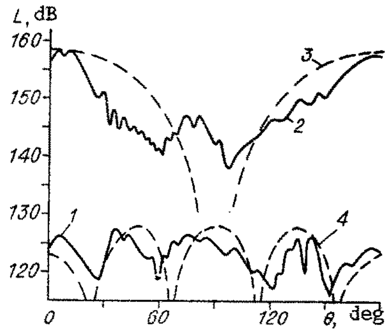

Fig. 4

when the dipoles are oppositely directed and

$$\cos\theta\cdot\cos\left(\frac{1}{2}kl\cos\theta\right) \tag{2.3}$$

when the dipole moments have the same direction. Here, k is the wave number; ℓ is the distance between the dipoles. Line 3 in Fig. 4 shows Eqs. (2.2) corresponding to the conditions in diagram 2. It can be seen from a comparison of curves 2 and 3 that the empirical dependence is narrower and has a small central peak. This is apparently connected with the multiplicity of the interacting dipoles which form the field of the discrete tone of the jets.

The multilobed radiation diagram seen when m = 0 can be interpreted as a manifestation of interference of the radiation of cophasal multipole sources located on the jet axis. This interpretation is based on the axial symmetry of the perturbations associated with the given mode. As an illustration, curve 4 shows the theoretical indicatrix for the discharge conditions corresponding to diagram 1. This result agrees satisfactorily with the experimental relation.

3. We measured the frequency and amplitude of the discrete tone and its direction in the plane perpendicular to the jet axis for a system in which a single jet was discharged parallel to a plane rigid shield. It was found that, as in the previous system, in this case the wave structure of the jet undergoes axisymmetric and flexural vibrations (photographs 4 and 5 in Fig. 1), the conditional boundary of these oscillations being $M_t = 1.25$. The photographs were taken with the following discharge parameters: 4) $M_a = 1$, n = 1.33, $\overline{S} = 1.8$; 5) $M_a = 1$, n = 1.51, $\overline{S} = 3$. The measured frequencies of the discrete tone are described by Eqs. (2.1). The character of change in the amplitude of the discrete tone as a function of the parameters M_a, n, and $\overline{S}$ is similar to the case of a two-jet system (Fig. 2b). However, in the present case, as the distance $\overline{S}$ it is necessary to take twice the distance between the nozzle axis and the shield. The underexpansion intervals coincide for the one- and two-jet systems. Given the same set of discharge parameters, the direction of the radiation of the discrete tone in the plane of the moving sensor is characterized by the same radiation indicatrices (being of the type shown in Fig. 4). The angle θ is reckoned from a normal drawn to the shield surface and crossing the jet axis.

The data presented here makes it possible to conclude that the same mechanism is responsible for sustaining the oscillations in the two-jet systems that we are comparing. The parallel shield functions as an acoustic mirror for the jet. The actual jet interacts acoustically with its image in the surface of the shield, and this interaction determines the amplitude-frequency characteristics of the oscillations in the jet-shield system. On the other hand, the fact that the characteristics for two different jet systems agree is evidence of the predominantly acoustic nature of the interaction between the jets.

4. For single free supersonic underexpanded jets, the possibility of excitation of oscillations depends to a significant extent on the acoustic situation in the region near the beginning of the jet, since the latter determines the efficiency of the feedback loop

in an oscillating jet system [19]. To realize positive feedback, the phase condition for the waves of the discrete tone must be such that acoustic pressure pulsations promote the development of hydrodynamic perturbations in the jet, and these perturbations in turn generate sound waves. Considering the fact that the root of a jet in a two-jet system is subjected both to natural sound waves and waves associated with the discrete tone of the companion jet [3], it can be expected that the phase relations in the feedback loop will depend on the distance between the jets (or between the jet and its acoustic image). The characteristic dimensions determining the length of the loop in a two-jet system are:

– the distance from the effective source of the sound waves to the point of closure of the loop for each jet (dimension A);

– the distance from the effective source of one jet to the point of closure of the loop of the other jet (dimension B);

– the distance between the boundaries of the jets (dimension C).

Here, the natural scale of distance will be the wavelength of the discrete tone λ_d. The distance between the boundaries of the jets is determined as $C = S - d_t$. Data on the position of the point of closure of the feedback loop in subcritical underexpanded jets was presented in [20] as a function of M_a and n. The position of the effective source of sound at the frequency of the discrete tone was determined from the phase shift of the mutual correlation function of the signals of the two pressure-pulsation sensors located along the jet. Comparison of the results of measurements with schlieren photographs of the wave structure of the jets showed that the effective source is located in the region between the fourth and eighth cells of the wave structure, and its position in each specific case is determined by the amplitude of the discrete tone. This is fully consistent with the results in [14, 19]. An analysis of the correlation measurements made it possible to generalize data and link the distance from the nozzle edge to the sound source h_s (expressed through the number of wave-structure cells within this interval) with the intensity of the discrete tone in the approximate relation

$$h_s \simeq 8 - 0{,}11\Delta L \qquad (4.1)$$

(ΔL is the amount by which the intensity of the tone exceeds the level of the noise in the same spectral range, dB). The distance from the nozzle edge to the effective source $\bar{\ell}_s$, expressed in nozzle diameters, was found from schlieren photographs of the flow field with the use of the measured quantity ΔL and expression (4.1). The value of A was determined as the difference $\bar{\ell}_s - \bar{\ell}_a$ ($\bar{\ell}_a$ is the distance from the nozzle edge to the point of closure of the feedback loop from [20]), while B was determined from the relation $B = (A^2 + C^2)^{1/2}$.

Figure 5 shows the positions of local maxima of the intensity of the discrete tone in the plane of the coordinates: distance $(B - A)/\lambda_d$ (x-axis) – distance C/λ_d (y-axis). The graph shows our data for both jet systems for modes B and C (points 1 and 2), as well as [3] (points 3 and 4), [7, 9], and [10] (points 5 and 6) for mode B. It is evident that the experimental points are grouped in hatched regions centered at points of the phase plane described by the relations

$$(B - A)/\lambda_d = (R - 1),$$
$$C/\lambda_d = (N - 1/2); \qquad (4.2)$$
$$(B - A)/\lambda_d = (R - 1/2),\ C/\lambda_d = N\ (R,\ N = 1,\ 2,\ 3,\ \ldots). \qquad (4.3)$$

The positions of the maxima of oscillation intensity with m = ±1 (symmetric relative to the plane of separation of the nozzles) fall within regions with origins determined by (4.2). The positions of the maxima at m = ±1 (asymmetric relative to the plane of separation) are grouped in regions with an origin determined by Eq. (4.3). An oscillation maxima is seen at m = 0 only in a square with its center at the point (0.5; 0). This pattern can be explained as follows. When flexural oscillations of the wave structure of a jet occur, the acoustic waves reinforce one another in the region of the root of the jets only in the case when the boundary between them is a multiple of the period of oscillation. On the other hand, the resulting field of interacting dipoles with oppositely directed moments lies in the plane of the jets only when – in accordance with (2.2) – the distance between them is a multiple of an odd number of oscillation half-periods.

One consequence of this is the need to satisfy conditions (4.2), or at least the conditions prevailing within the hatched areas in Fig. 5. With an increase in the distance be-

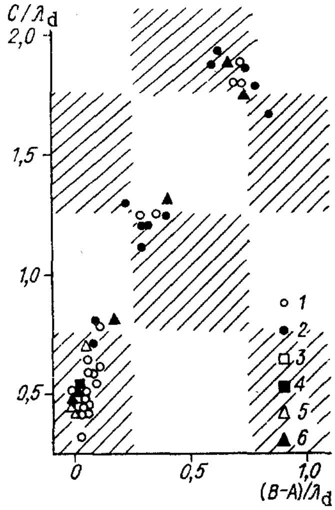

Fig. 5

tween the jets to a value which is a multiple of the wavelength λ_d, the resulting field of dipoles can lie within the plane of the jets only when [in accordance with (2.3)] the dipoles have the same direction, i.e., the oscillations of the wave structure are asymmetric relative to the plane of separation of the jets. Here, for the acoustic waves arriving at the roots of the jets to reinforce one another, they must be phase-shifted by an amount which is a multiple of an odd number of half-periods. This means that condition (4.3) or conditions close to (4.3) must be satisfied. The phase conditions for the existence of positive feedback in the oscillating jet system are violated in the unhatched regions of Fig. 5. The considerably greater intensity of the maxima lying in the region with its center at the point (0.5; 0) compared to the other maxima is due on the one hand to their proximity to the point corresponding to the optimum phase conditions for feedback and, on the other hand, to the more substantial acoustic interaction of the jets at shorter distances.

With excitation of the axisymmetric mode (m = 0), the effective dipoles are parallel to one another and the axes of the jets and have moments of the same direction. In this case, the diagram of the radiation of the discrete tone in the plane of the jets is described by the function [18]

$$\cos\varphi \cdot \cos\left(\frac{1}{2}\,kl\sin\varphi\right) \tag{4.4}$$

(φ is an angle reckoned from the jet axis). When $\ell = \lambda_d/2$, the radiation is directed upstream and downstream in two lobes. In the plane perpendicular to the jet axis, the radiation of each jet has axial symmetry. This ensures adequate efficiency for the feedback loop of the oscillating jet system if the jets are close enough together and if the difference in the phases of the waves arriving at the root of the jet is a multiple of the period of oscillation. An increase in the distance between nozzles is accompanied by a decrease in the extent of mutual acoustic radiation of the jets. Also, in accordance with (4.4), with an increase in the distance above $\lambda_d/2$, additional lobes that are not directed upstream appear in the direction diagram. Both of these factors reduce the efficiency of the feedback loop outside the region with its center at point (0.5; 0). Thus, Eqs. (4.2) can be used to describe the phase conditions for the existence of oscillations with m = 0 and small nozzle spacings.

The validity of conditions (4.2)-(4.3) was checked experimentally by moving the nozzles lengthwise relative to one another. Here, the discharge conditions corresponded to regions with centers at the points (0.5; 0) and (1.0; 0.5) (Fig. 5) and m = ±1. It turns out that the discrete tone is completely suppressed when the nozzles are displaced by the amount $\approx\lambda_d/2$, since this is accompanied by a shift in the phase conditions in the unhatched regions of the phase plane. In fact, with displacement of the nozzles by $\lambda_d/4$, the intensity of the discrete tone decreases to a level which is close to the level for two acoustically independent jets (the dot-dash lines in Fig. 2a). However, the original level of oscillation is restored with further nozzle displacement. An analysis of the stroboscopic schlieren photographs showed that, in this case, there is a 180° change in the phase of oscillation of the wave structure of one of the jets. This ensures satisfaction of the phase conditions in the initial hatched area. Thus, displacement of the nozzles by $\lambda_d/2$ does not change the oscillations from being symmetric to being asymmetric relative to the plane of separation, or vice versa. The oscillations of the wave structure always correspond to a specific half-wave parity of the distance between the boundaries of the jet. This confirms the validity of Eqs. (4.2) and (4.3).

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